3-6 Derivatives of Parametric Functions

Learning Objectives:

I can find the derivatives of parametrically defined curve

I can find the second derivative of a parametrically defined curve

I can write the equation of the tangent line to a parametrically defined curve

I can find the lowest, highest, leftmost, and rightmost points of parametrically defined curve

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Derivatives of Parametrically Defined Functions

x(t) y(t)

$$\frac{dx}{dt} = x'(t) \qquad \frac{dy}{dt} = y'(t) \qquad \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

provided $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dy}{dx}$ all exist and $\frac{dx}{dt} \neq 0$

Ex1. Given the parametric equations

$x = t^2 \qquad y = e^{2t}$

a.) Find $\frac{dy}{dx}$ in terms of t.

$\frac{dx}{dt}, \frac{dy}{dt}$

$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 2e^{2t}$

$$\frac{dy}{dx} = \frac{2e^{2t}}{2t} = \boxed{\frac{e^{2t}}{t}}$$

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$x=t^2$ $y=e^{2t}$

b.) Write the equation of the tangent line to the curve at (4, e⁴)

$$\frac{dy}{dx}=\frac{e^{2t}}{t}$$

$$y-y_1=m(x-x_1)$$

$4=t^2$

$t=\pm 2$

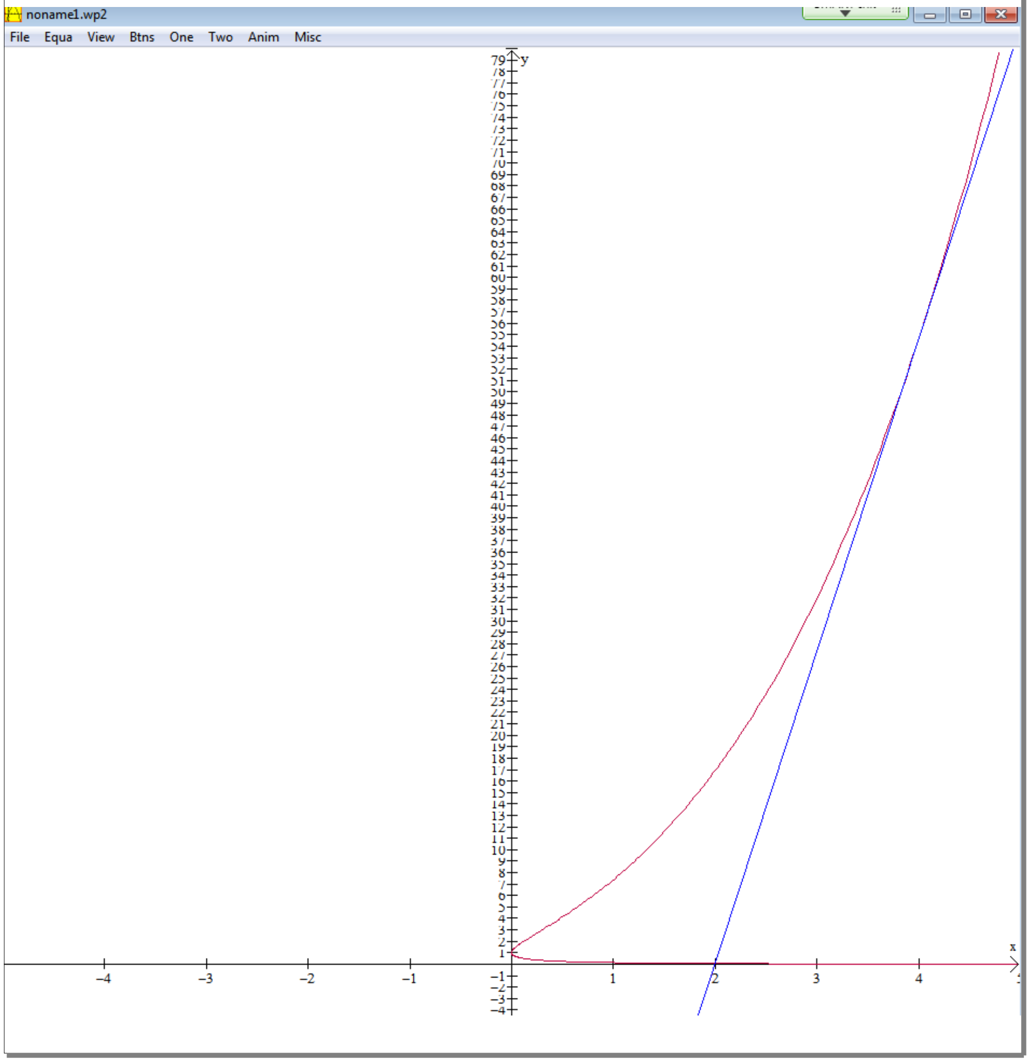
$y=e^{2t}$

$e^4=$

$\underline{t=2}$

$$\frac{dy}{dx}(t=2)=\frac{e^4}{2}$$

$$\boxed{y-e^4=\frac{e^4}{2}(x-4)}$$



Second Derivatives of Parametrically Defined Functions

x(t) y(t)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(dy/dx\right)}{dx/dt}$$

provided $\frac{dx}{dt}$, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ all exist and $\frac{dx}{dt} \neq 0$

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$$\frac{f'g - fg'}{g^2}$$

$x = t^2$ $y = e^{2t}$

$$\frac{dy}{dx} = \frac{\overset{f}{e^{2t}}}{\underset{g}{t}}$$

c.) Find $\frac{d^2y}{dx^2}$ in terms of t.

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{\frac{2e^{2t}\cdot t - e^{2t}\cdot 1}{t^2}}{\frac{2t}{1}}$$

$$\boxed{= \frac{2te^{2t} - e^{2t}}{2t^3}}$$

@ $(4, e^4)$ $t=2$

$$\frac{dy}{dx} = \frac{e^4}{2} = +$$

$$\frac{d^2y}{dx^2} = \frac{2te^{2t} - e^{2t}}{2t^3} = \frac{4e^4 - e^4}{2(2)^3}$$

$$= \frac{3e^4}{16} = +$$

inc & con up

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Ex3. The prolate cycloid given by

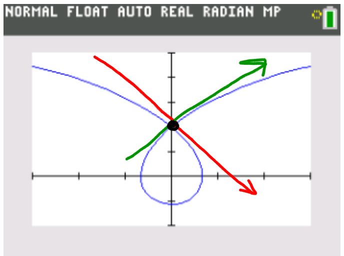
$x = 2t - \pi\sin(t)$ $dx/dt = 2 - \pi\cos t$

$y = 2 - \pi\cos(t)$ $dy/dt = \pi\sin t$

crosses itself at the point (0,2). Find the equation of both tangent lines at that point.

$y - y_1 = m(x - x_1)$

$$\frac{dy}{dx} = \frac{\pi\sin t}{2 - \pi\cos t}$$



$2 = 2 - \pi\cos t$

$0 = -\pi\cos t$

$0 = \cos t$

$t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$-\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, \ldots$

$t = \pi/2$

$$\frac{dy}{dx} = \frac{\pi\sin(\pi/2)}{2 - \pi\cos(\frac{\pi}{2})} = \frac{\pi}{2}$$

$y - 2 = \frac{\pi}{2}(x - 0)$

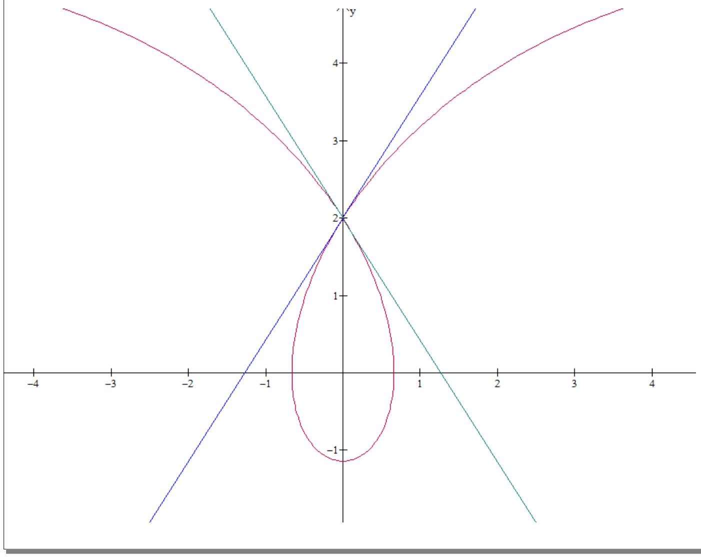
$\boxed{y = \frac{\pi}{2}x + 2}$

$t = -\frac{\pi}{2}$

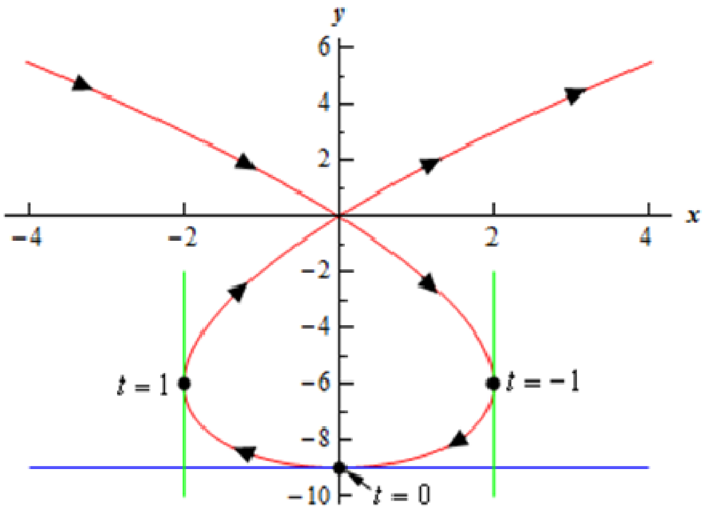
$$\frac{dy}{dx} = \frac{\pi\sin(-\frac{\pi}{2})}{2 - \pi\cos(\frac{\pi}{2})} = -\frac{\pi}{2}$$

$y - 2 = -\frac{\pi}{2}(x - 0)$

$\boxed{y = -\frac{\pi}{2}x + 2}$



Extrema Parametric Curves

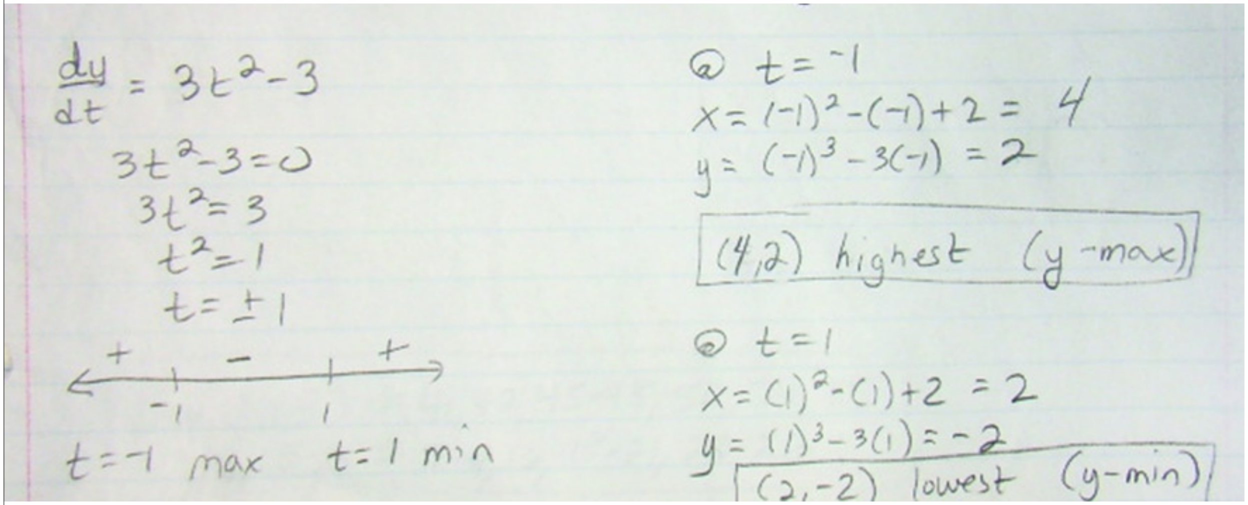


Ex4. Given the parametric equations

$$x = t^2 - t + 2$$
$$y = t^3 - 3t$$

Find the following points if they exist:

a.) Rightmost b.) Leftmost
c.) Lowest d.) Highest



$\frac{dy}{dt} = 3t^2 - 3$

$3t^2 - 3 = 0$
$3t^2 = 3$
$t^2 = 1$
$t = \pm 1$

$+ \quad - \quad +$

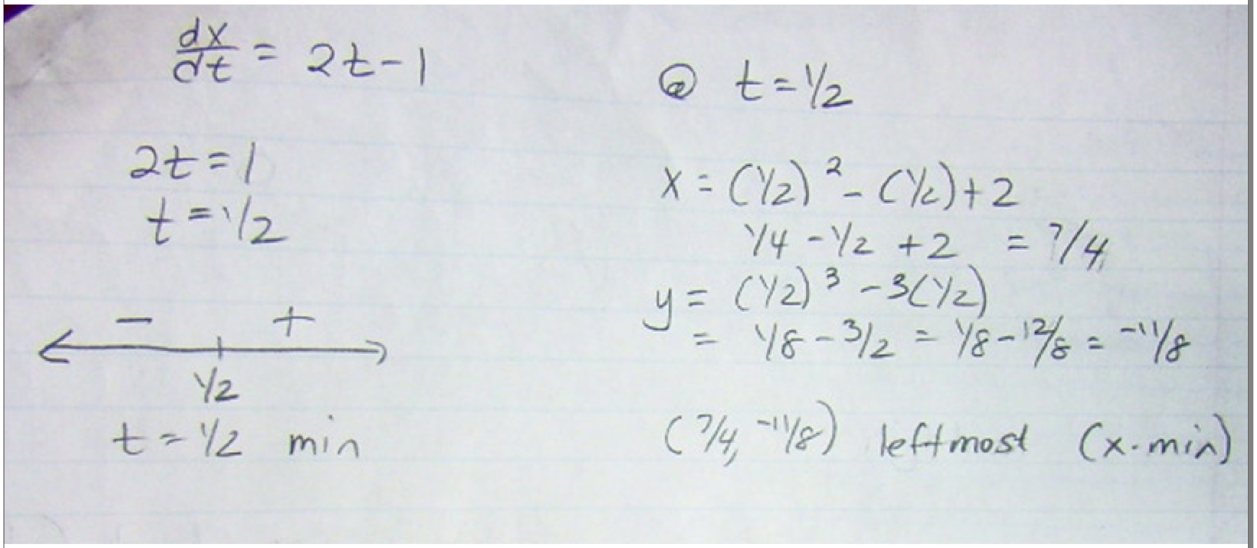
$-1 \quad 1$

$t = -1$ max $t = 1$ min

@ $t = -1$
$x = (-1)^2 - (-1) + 2 = 4$
$y = (-1)^3 - 3(-1) = 2$

$\boxed{(4,2) \text{ highest } (y\text{-max})}$

@ $t = 1$
$x = (1)^2 - (1) + 2 = 2$
$y = (1)^3 - 3(1) = -2$
$\boxed{(2,-2) \text{ lowest } (y\text{-min})}$



$\frac{dx}{dt} = 2t - 1$

$2t = 1$
$t = 1/2$

$- \quad +$

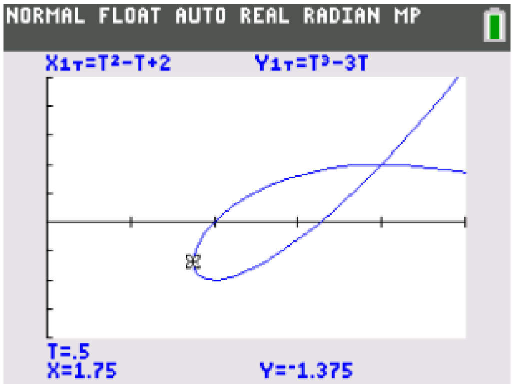
$1/2$

$t = 1/2$ min

@ $t = 1/2$

$x = (1/2)^2 - (1/2) + 2$
$1/4 - 1/2 + 2 = 7/4$
$y = (1/2)^3 - 3(1/2)$
$= 1/8 - 3/2 = 1/8 - 12/8 = -11/8$

$(7/4, -11/8)$ leftmost $(x\text{-min})$

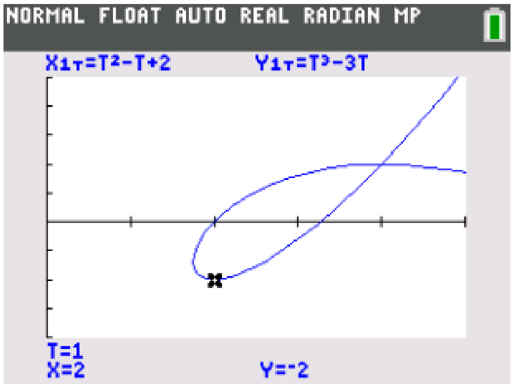


NORMAL FLOAT AUTO REAL RADIAN MP

$X_{1T}=T^2-T+2$ $Y_{1T}=T^3-3T$

$T=.5$

$X=1.75$ $Y=-1.375$

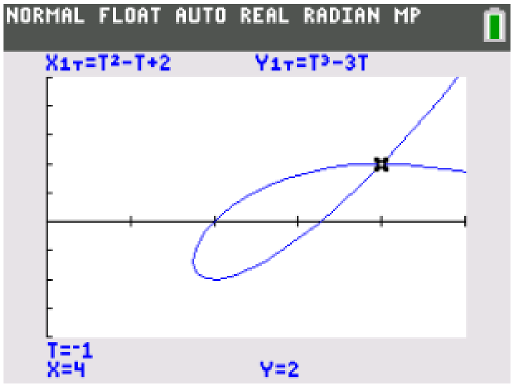


NORMAL FLOAT AUTO REAL RADIAN MP

$X_{1T}=T^2-T+2$ $Y_{1T}=T^3-3T$

$T=1$

$X=2$ $Y=-2$



NORMAL FLOAT AUTO REAL RADIAN MP

$X_{1T}=T^2-T+2$ $Y_{1T}=T^3-3T$

$T=-1$

$X=4$ $Y=2$

Homework

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